Modeling for Frequency Characteristics of Oil-paper Composite Insulation Based on Fractional Calculus

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Oil-paper composite insulation is an important part of insulation of the transformer equipments and its circuit mode plays an important role in researching influence of the frequency-dependent dielectric constant on transformer model. From the basic theory of dielectric, fractional theory is used to improve the present integer-order circuit model of oil-paper composite insulation and fractional circuit model of oil-paper composite insulation is proposed preliminarily.Then, the proposed fractional circuit model has been vertified by dielectric spectroscopy of oil-paper composite insulation. Finally, by comparing the fitting residuals of the integer-order model and fractional model and using proposed fractional circuit model to fit polarity reversal voltage response of oil-paper composite insulation, the conclusion can be drawn that fractional model in this paper has feasibility and veracity.

***Index Terms*—Dielectric constant, oil-paper insulation, fractional calculus, numerical computation**

I. INTRODUCTION

OIL-PAPER COMPOSITE INSULATION is a kind of composite dielectrics that are used for the insulation of the transformer type electrical equipment. Debye theory has been accepted widely. Then, traditional deby etheory has been improved [1]. The Maxwell-Wagner model for oil-paper composite insulation system can't explain precisely experimental data [2]. A broadband mathematical model for oil-paper composite insulation system was established which is called Havriliak-Negami model with two relaxation branchs [3]. But circuit model for oil-paper composite insulation system has been not brought up. By making use of the related theory of fractional calculus, this paper based on the theory of dielectric proposed a more concise fractional order circuit model.

II. THEORY OF FRACTIONAL CALCULUS

In mathematics, the fractional calculus has not been unified defined in time domain so far. Riemann-Liouville formula used in this paper is defined as

$$ {}_a^R I_t^{\nu} s(t) = \frac{1}{\Gamma(-\nu)} \int_a^t (t-\tau)^{-\nu-1} s(\tau)\, d\tau, \nu < 0 \quad (1)$$

$$ {}_a^R D_t^{\nu} s(t) = \begin{cases} \dfrac{d^n s}{dt^n}, & \nu = n \in \mathbb{N} \\ \dfrac{d^n}{dt^n} \dfrac{1}{\Gamma(n-\nu)} \displaystyle\int_a^t \frac{s(\tau)}{(t-\tau)^{\nu-n+1}} d\tau, & (0 \le n-1 < \nu < n) \end{cases} \quad (2)$$

Fractional component is also known as constant phase element(CPE)[4]. Fractional order capacitance element can be represented as

$$ i = C_\alpha\, d^\alpha u(t) / d^\alpha t \quad (3)$$

Admittance is indicated respectively as follows

$$ Y = C_\alpha (j\omega)^\alpha = C_\alpha (s)^\alpha \quad (4)$$

III. FRACTIONAL ORDER CIRCUIT MODEL FOR OIL-PAPER COMPOSITE INSULATION

According to the following formula in [5]

$$ i(t) = G_g u(t) + C_g \frac{du(t)}{dt} + C_0 \frac{d}{dt}\left\{ \int_0^t h(\tau) u(t-\tau) d\tau \right\} \quad (5)$$

Considering the formula (6) in [1]

$$ F^*(j\omega) = \chi^*(j\omega) = A \frac{1}{(j\omega)^\alpha}, \quad 0 < \alpha < 1 \quad (6)$$

Formula (7) can be drawn by fourier inverse transformation

$$ f(t) = A \cdot \frac{t^{\alpha-1}}{\Gamma(\alpha)} = A \cdot \frac{1}{\Gamma(\alpha)} \cdot \frac{1}{t^\nu}, \nu = 1-\alpha \quad (7)$$

It can be simplified to formula (8)

$$ f(t) = K \cdot \frac{1}{t^\nu}, 0 < \nu < 1 \quad (8)$$

Here $K = A \cdot \dfrac{1}{\Gamma(\alpha)}$ and it is a constant.

Applying (8) to (5), equation (5) can be derived as

$$ i(t) = G_g u(t) + C_g \frac{du(t)}{dt} + KC_0 \frac{d}{dt}\left\{ \int_0^t \frac{u(\tau)}{(t-\tau)^\nu} d\tau \right\}, 0 < \nu < 1 \quad (9)$$

As Riemann-Liouville definition if $n = 1$, $a = 0$, it can be writed as follow

$$ \frac{d^\nu u(t)}{dt^\nu} = \frac{1}{\Gamma(1-\nu)} \cdot \frac{d}{dt}\left\{ \int_0^t \frac{u(\tau)}{(t-\tau)^\nu} d\tau \right\} \quad (10)$$

Taking count of formula (10), equation (9) can be finally expressed as

$$ i(t) = G_g u(t) + C_g \frac{du(t)}{dt} + C_\nu \frac{d^\nu u(t)}{dt^\nu}, \quad 0 < \nu < 1 \quad (11)$$

where $C_\nu = KC_0 \cdot \Gamma(1-\nu)$.

The corresponding circuit model is as figure 1.

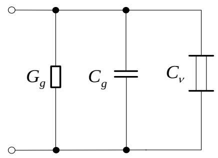

Fig.1. Fractional orderequivalent circuit model.

Assuming that the transformer oil and insulation board are homogeneous medium respectively,oil-paper board can be seen as two series of homogeneous medium.So improved Maxwell-Wagner model is put forward as figure 2.

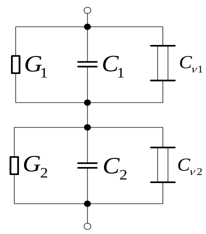

Fig.2. Improved Maxwell-Wagner model.

IV. VALIDATION OF IMPROVED MAXWELL-WAGNER MODEL

To verify the correctness of improved Maxwell-Wagner model, the measurement data of dielectric response of oil-paper which was taken from [3] was fitted by using genetic algorithm. The fitting results are shown in figure 3.

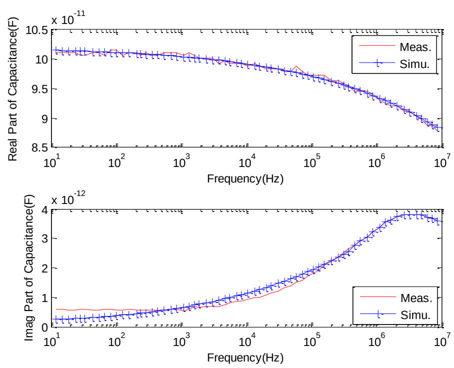

Fig. 3. Fitting result of improved Maxwell-Wagner model.

V.THE COMPARISONS OF MODEL

The superiority of improved Maxwell-Wagner model can be seen explicitely by comparisons with traditional Maxwell-Wagner model [2] and extended Debye model [6]. Useing genetic algorithm the same measurement data has been fitted as figure 4. The green line and blue one and balck one present respectively traditional Maxwell-Wagner model and improved Maxwell-Wagner model and extended Debye model. The fit residuals of this three models are shown as table 1. It can be seen from table 1 that the fit residual of improved Maxwell-Wagner model is the minimum among this three models. So the conclusion can be drawn that improved Maxwell-Wagner model is more accurate than other models.

TABLE I
THE FIT RESIDUALS OF THREE MODELS

Types of model	Fit Residuals
Maxwell-Wagner	3.436e-03
Improved Maxwell-Wagner	2.732e-05
Extended Debye	3.016e-04

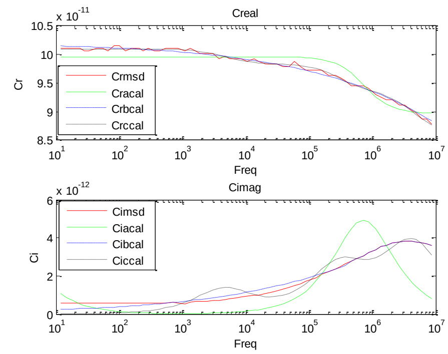

Fig. 4. Comparisons of traditional Maxwell-Wagner model and improved Maxwell-Wagner model and extended Debye model.

VI. SIMPLE APPLICATION

In order to verify practicability of improved Maxwell-Wagner model , the measurement data[2] of dielectric response of oil-paper composite insulation under polarity reversal voltage was compared with results of numerical simulation[6] of mode as figure 5.

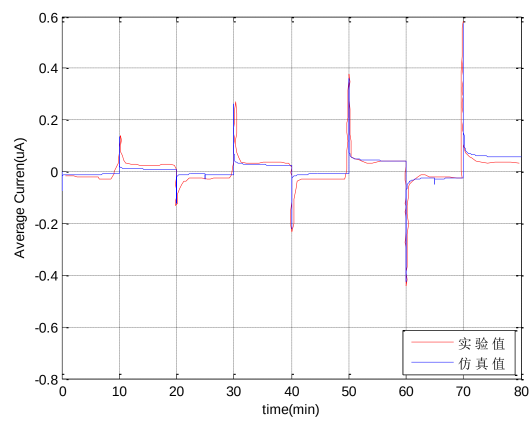

Fig.5. Improved Maxwell - Wagner model fitting polarity reversal voltage response.

VII. REFERENCES

[1] A K. Jonscher, *Dielectric Relaxation in Solid*, Chelsea Dielectrics Press, 1983.

[2] Wang Yonghong, Wei Xinlao and Chen Qingguo, "Characteristics of Oil-pressboard Insulation Under Polarity Reversal Voltage," in Proceedings of the CSEE, 2012, 32(19):161-168.

[3] Zheng Yiming and Wang Zanji, "Study on Broadband Loss Characteristics of Oil-immersed Papers for Fast Transient Modeling of Power Transformer", IEEE Transactions on Dielectrics and Electrical Insulation . Vol. 20, No. 2, April 2013.

[4] Karabi Biswas, Siddhartha Sen, and Pranab Kumar DuttaS,"Realization of a Constant Phase Element and Its Performance Study in a Differentiator Circuit", IEEE Transactions on Circuits and Systems. Vol. 53, No. 9, Seprember 2013.

[5] Birlasekaran, Y Xingzhou. Relaxation Studies on Power Equipment. IEEE Transactions on Dielectrics and Electrical Insulation, 2003, 10(6):1061-1077.

[6] Xue Dingyu, Chen Yangquan. Control of MATLAB to solve the math problem. Beijin: Tsinghua University Press, 2007.